



# Design of Snow Plowing and Deicing Routes for Urban Winter Viability : Addressing Actual Operational Constraints

- **Olivier Quirion-Blais**
- Ph. D. student
- Polytechnique Montréal
- [olivier.quirion-blais@polymtl.ca](mailto:olivier.quirion-blais@polymtl.ca)

Martin Trépanier, Ph. D., P. Eng.

André Langevin, Ph. D.

Professor

Polytechnique Montréal



POLYTECHNIQUE  
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## **0. CONTENT**

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### **1. Introduction**

### **2. Methodology**

**Case Study**

**Constraints Raised**

**Mathematical Programming**

### **3. Implementation**

**Solving the Mathematical Formulation**

### **4. Conclusion**

**Metaheuristic**

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## 1. INTRODUCTION

### Operational Problems in Winter Viability:

- Work sectors design
- Vehicle routing (snow plow, spreader, snow removing, snow hauling)
- Crew and fleet sizing
- Subcontractor
- Budget planning
- Scheduling



## 1. TYPE OF SOLVING APPROACHES

### Simulations

- Tucker and Clohan, 1979

Precise  
results over  
time  
Case specific

### Mathematical programming

- Tagmouti et al., 2007

Optimality  
Small  
instances

### Heuristics

- Lemieux and Campagna, 1984
- Perrier et al., 2008

Quick results  
Depends on  
the case

### Metaheuristics

- Handa et al., 2005 and 2006
- Omer, 2007
- Tagmouti et al., 2010

Generalist  
Long  
development  
time

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Used to assess  
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### Metaheuristics

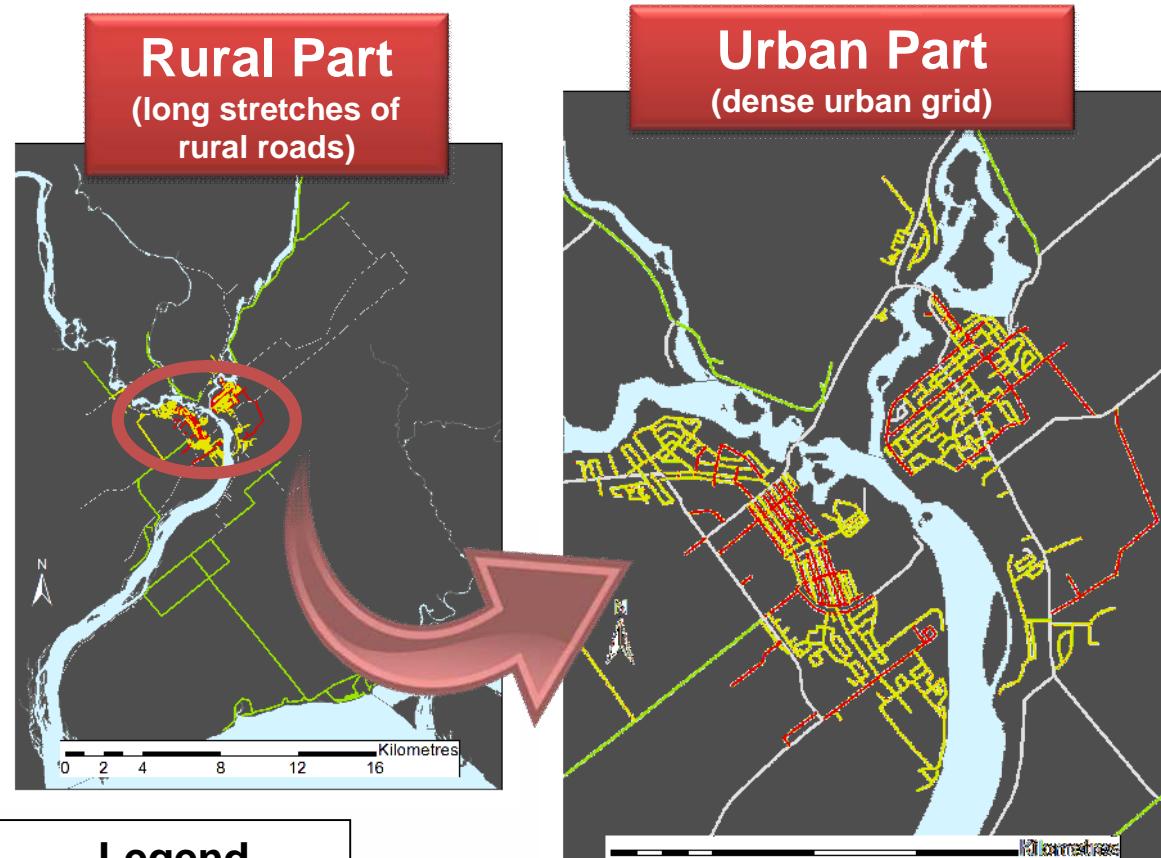
- Handa et al., 2005 and 2006
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Long  
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## 2. CASE STUDY

### The case of Dolbeau-Mistassini

Small town in northern Quebec :  
• 14,000 inhabitants  
• 8 vehicles  
• 260 km of roadway, 3 classes of priority  
• 1880 directed street segments, 44 undirected  
• 733 nodes (intersections)



## 2. CONSTRAINTS RAISED

### Workload Balance



### Network Hierarchy



### Road/Vehicle Dependency



*Image source :*  
<http://www.flickr.com/photos/jamescastle/4389696603>

### Turn Restrictions



### Partial Area Coverage



Other constraints  
not considered for  
this study :

- Vehicle capacity
- Tandem synchronization
- Recurring service

## 2. MATHEMATICAL FORMULATION

### Objective Function

$$\text{Min} \left( \sum_{p=1}^P (TMAX^P M^P) + T^{tot} M^{tot} \right.$$
$$\left. + \sum_{p=1}^{P+1} \sum_{v=1}^V \sum_{k=1}^K \sum_{i \in A} \sum_{j | x_{ij}^{kvp} \exists} f_{ij} (x_{ij}^{kvp} + y_{ij}^{kvp}) \right)$$



## 2. MATHEMATICAL PROGRAMMING

### Constraints

$$2. TMAX^P \geq t^{vp}$$



$$p = \{1 \dots P+1\}$$

$$3. t^{vp} = \sum_{k=1}^K \sum_{i \in A} \sum_{j \in A} (x_{ij}^{kvp} tp_i^v + y_{ij}^{kvp} ts_i^v) \quad p = \{1 \dots P+1\}, v = \{1 \dots V\}$$

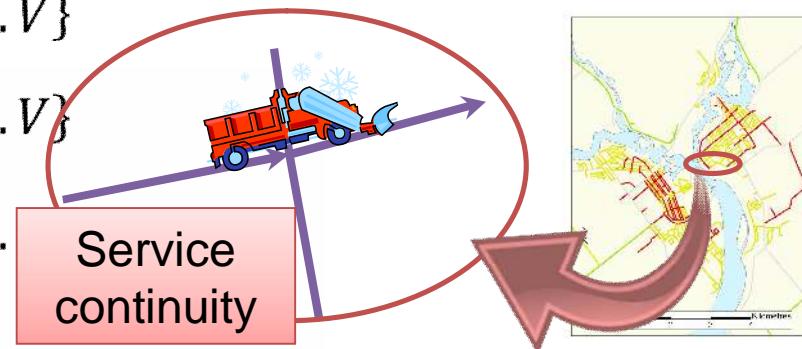
$$4. \sum_{p=1}^{P+1} \sum_{j \in A | x_{ij}^{kvp} \neq 0} (x_{ij}^{kvp} + y_{ij}^{kvp}) = \sum_{p=1}^{P+1} \sum_{l \in A | x_{li}^{k+1,vp} \neq 0} (x_{li}^{k+1,vp} + y_{li}^{k+1,vp})$$

$k = \{2 \dots K\}, v = \{1 \dots V\}, i \in A \cup A_{end} \cup a_{init}$

$$5. \sum_{p=1}^{P+1} \sum_{j \in d} x_{ia_{init}}^{1vp} = 1 \quad v = \{1 \dots V\}$$

$$6. \sum_{p=1}^{P+1} \sum_{i \in A} \sum_{j | x_{ij}^{1vp} \neq 0} x_{ij}^{1vp} = 0 \quad v = \{1 \dots V\}$$

$$7. \sum_{k=1}^K \sum_{i \in A_{end}} \sum_{j \in A} x_{ij}^{kvp+1} = 1 \quad v = \{1 \dots V\}$$



## 2. MATHEMATICAL PROGRAMMING

### Constraints

$$8. \sum_{j|x_{ij}^{kvp}} (x_{ij}^{kvp} + y_{ij}^{kvp}) \leq \sum_{p*=p}^{P+1} \sum_{l|x_{li}^{kvp}} (x_{li}^{k+1,vp*} + y_{li}^{k+1,vp*})$$

$k = \{2 \dots K\}, p = \{1 \dots P, P + 1\}, i \in A \cup a_{init} \cup A_{end}$



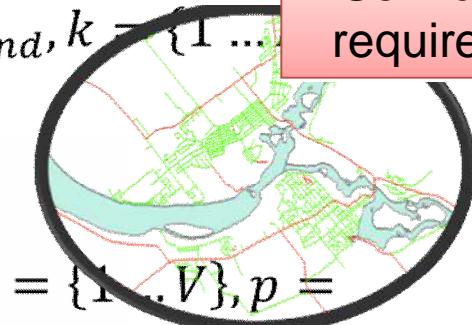
Hierarchy

$$9. \sum_{k=1}^K \sum_{v=1}^V \sum_{p*=1}^p \sum_{j|y_{ij}^{kvp*}} y_{ij}^{kvp*} = 1 \quad i \in A_{ToBeServiced}^p, p = \{1 \dots P\}$$

$$10. x_{ij}^{kvp} \in \{0,1\} \quad i \in A \cup A_{end}, j \in A \cup a_{init} \cup A_{end}, k = \{1 \dots V\}, p = \{1 \dots P, P + 1\}$$

$$11. y_{ij}^{kvp} \in \{0,1\}$$

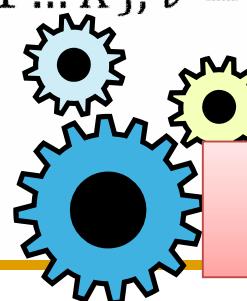
$$i \in A \cup A_{end}, j \in A \cup a_{init} \cup A_{fin}, k = \{1 \dots K\}, v = \{1 \dots V\}, p = \{1 \dots P, P + 1\}$$



Service required

$$12. t^{vp} \geq 0 \quad v = \{1 \dots V\}, p = \{1 \dots P, P + 1\}$$

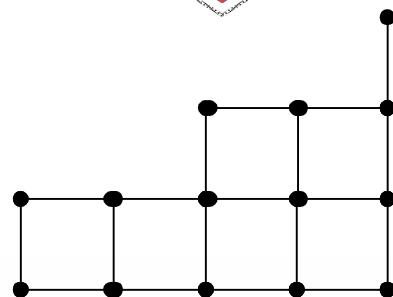
$$13. TMAX^p \geq 0 \quad p = \{1 \dots P, P + 1\}$$



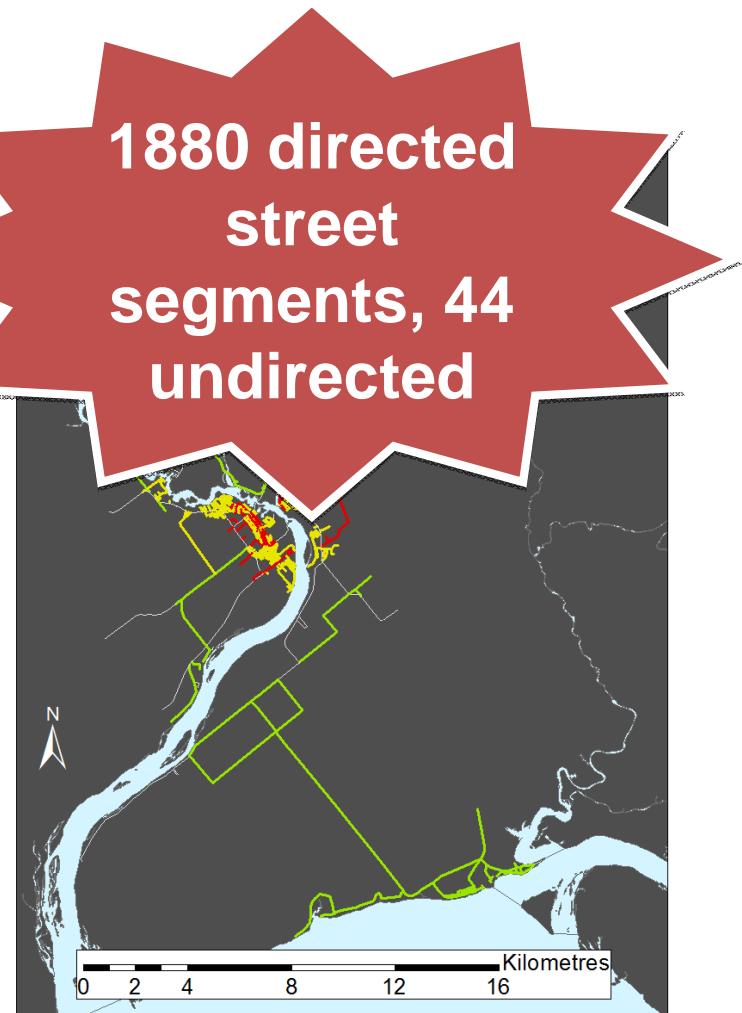
Positivity and binary

### 3. SOLVING THE MATHEMATICAL FORMULATION

# 56 directed street segments



1880 directed  
street  
segments, 44  
undirected



### 3. SOLVING REAL CASE STUDIES

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#### Mathematical formulation

- Long solving time
- Limited to 28 street segments



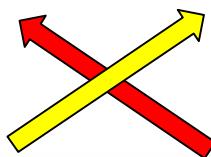
#### Metaheuristic

- Relatively short solving time (few hours)
- Big instances



## 4. Metaheuristic

Metaheuristic : search process guided by an objective function that uses simple arc exchanges (operators) in order to improve the global solution.



The improvement of  
the solution is  
measured with the  
objective function



## 4. Acknowledgments

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**CRSNG  
NSERC**

*Fonds de recherche  
sur la nature  
et les technologies*

**Québec**



Contact : [olivier.quirion-blais@polymtl.ca](mailto:olivier.quirion-blais@polymtl.ca)