DESIGN OF SNOW PLOWING AND DEICING ROUTES FOR URBAN WINTER VIABILITY: ADDRESSING ACTUAL OPERATIONAL CONSTRAINTS

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ABSTRACT

The planning of vehicle routes for winter road maintenance is a time consuming task. Traditionally it is done by hand based on the experience and knowledge of the managers. The road networks getting larger, the expenses growing, and the climate changing, new ways of designing snow plow routes for better efficiency are required. To help planners, scientists are developing new mathematical methodologies to implement into planning tools. This article presents an exact model to address the operational constraints of snow plowing and deicing route planning, but also shows its limitations in view of a real world problem.

CONCEPTION DE TOURNÉES DE DÉNEIGEMENT ET D'ÉPANDAGE EN VIABILITÉ HIVERNALE URBAINE: CONSIDÉRATION DES CONTRAINTES OPÉRATIONNELLES ACTUELLES

RÉSUMÉ

La planification des parcours de déneigement est une tâche ardue demandant beaucoup de temps. Traditionnellement, les parcours sont conçus manuellement sur la base des connaissances des gestionnaires. Mais voilà que dans certaines organisations, la connaissance se perd, les fonds sont limités et les changements climatiques font évoluer les besoins sur le terrain vers des outils d'aide à la décision. Pour aider les planificateurs, les scientifiques développent de nouveaux modèles mathématiques qui font partie intégrante de ces outils de confection de parcours. Cet article présente un modèle mathématique apte à proposer une résolution exacte du problème de déneigement et d'épandage dans un réseau urbain, tout en tenant compte des nombreuses contraintes opérationnelles rencontrées sur le terrain.

1. INTRODUCTION

In countries where harsh winter conditions occur, costly winter maintenance is required. In the major cities of the province of Quebec, Canada, the expenditures range between 5,000$ to 24,000$ by kilometres of roadway. Only for the provincial road network, this sums up to about 250 millions $ [1]. Around the world, it is more than 10 billions $ that are spent annually only to maintain the road networks safe and secure [2]. Furthermore, spreading road salt and abrasives is a significant source of pollution on the environment [3]. In the last years, since winter maintenance expenditure has increased a lot due to the cost of petrol and spreading materials, and because the road users have now higher expectation on the service quality [4], authorities are in search of new ways to cut expenditures.

New technologies such as GIS (geographic information systems) and RWIS (road weather information systems) provide more information from the field and new ways to process it. Ye, Shi and Strong [5] showed that these technologies could improve the management winter maintenance. One way of processing the information is to use mathematical algorithms that detremine new sets of plowing or salt spreading routes. This paper describes a new algorithm designed for snow plow routing. The first part presents the state of the art in snow plowing and salt spreading methods. Afterwards, a case study is presented, on which is based the mathematical model. Finally, the model is implemented on a very small network because of the resolution limitations. Hence, the design of a metaheuristics is next discussed to address the coverage of the complete road network.

2. BACKGROUND

2.1. Winter road maintenance problems

In general, winter maintenance problems have not been widely addressed in the literature. It can be explained by the complexity of the problem and the fact that it is often specific to each site. Several types of logistic problems can be encountered in winter maintenance:

- work sector design;
- routing for chemicals or abrasive spreading;
- routing for snow plowing;
- routing for loading snow into trucks;
- routing for hauling snow to disposal sites;
- vehicles, abrasives and chemicals depot location;
- crew and fleet sizing;
- budget planning;
- outsourcing to private contractors;
- crew, vehicles and maintenance scheduling.

Each of these problems has its specific constraints. Perrier et al. [6, 7, 8, and 9] have written an extensive four-part literature review on all these problems. Recently, Perrier et al. [10] extended the review in a book chapter dealing specifically with winter road spreading operations.

This paper focuses on the design of snow plowing routes. In this case, the major constraints to be considered are:

- Turn restrictions: U-turns must be avoided as much as possible for security and operational reasons. The same applies for left turns.
- Heterogeneous vehicles: some vehicles go faster than other. Therefore, these vehicles should be sent farther away.
- Street/vehicle dependency: narrow streets require smaller vehicles.
- Road network hierarchy: higher priority is given to commercial streets and in the neighborhood of police and fire station, hospital and school. Lower priority is given to rural streets;
- Workload balance: each vehicle should have about the same workload.
- Partial area coverage: some road segments are serviced by other authorities. These can be traversed, but don’t need to be serviced by the municipality’s vehicles.

The snow plow routing problem falls in the family of arc routing problems .For reasons of brevity, the literature review presented below focuses on works on snow plow routing.

2.2. Algorithms for snow plow routing

Four types of approaches have been put forward to design snow plow routes: Simulation, mathematical programming, heuristics, and metaheuristics.

2.2.1. Simulation and mathematical programming approaches

The earliest approach was simulation. Among the few decision tools that have been developed, Turk and Clohan [11] proposed an algorithm to help planners to improve snow plow routes. The algorithm is embedded into a simulation model which considers meteorological conditions, network initial conditions and plow trucks characteristics to assess the efficiency of the snow plow routes. The tool was tested within a case study on the town of Newington, Connecticut. The major drawback of simulation is that it usually takes a long time to develop and the models are often case specific.

The second type of approaches is mathematical programming. For example, Tagmouti et al. modeled the problem as a vehicle routing problem with time window (VRPTW)[12]. However, to use this formulation, they had to transform the network from an arc routing problem to a node routing problem using mathematical techniques. For their resolution, they also considered the capacity of the vehicles. They could solve instances of up to 40 required road segments. This shows that this kind of resolution can only be used for small instances as the number of variables and constraints increases exponentially. Indeed, in the case of snow plow routes design, this kind of method can hardly be used for real-life instances unless some constraints are relaxed. It is mainly used to demonstrate the complexity of the problem.

2.2.2. Heuristics

This third type of approaches consists in designing algorithm that will find good solution by taking advantage of the characteristics of the problem. Among these, the heuristics designed for snow plow routing can be classified in three categories: constructive heuristics, two-phase heuristics, and composite heuristics.

Constructive heuristics are the simplest as they are based on simple rules. For example, Lemieux and Campagna [13] proposed an algorithm that creates a subnetwork for each vehicle. While creating the subnetworks, they make sure that the depot is within the subnetwork and that both direction of all the road links are serviced by the same vehicle. Afterward, the streets of the each subnetwork are selected following given rules to create the plow routes. This type of method can produce good results in a short amount of time. The following two-phase heuristics have given better results.There are two types of two-phase heuristics: cluster first, route second and route first, cluster second. The former consists in partitioning the network among the vehicles available and then, create a route for each vehicle in the sector formed. An example of a cluster first, route second heuristic has been developed by Perrier et al. [14]. They applied their algorithm within a case study in the city of Dieppe, New Brunswick, Canada and they improved the plow routes used previously both in terms of total length and number of U-turns.

The final type of heuristic is the composite heuristics. This type mixes various techniques to build the routes. For example, Kandula and Wright [15] designed a three stage heuristic. In the first stage, a number of road links equal to the number of vehicles are selected following given rules. These road links are called seeds since the plow routes will be designed around them. Then, for each seed, road links are selected so that the route is continuous and in order to maximize the distance from the depot. The route is closed when no more road links can be found next to the current route or when the time limit is attained. It is closed by adding all the reverse direction of street in order to make sure that all vehicles cover both sides of the streets. An improvement stage is then performed by exchanging inside the routes and between the routes. This heuristic was implemented with data from the Indiana Department of Transportation (INDOT).

2.2.3. Metaheuristics

Metaheuristics are more complex methods made of an initial construction phase and an improving phase. This is the type of method that has been designed more recently and they often provide very good results. For example, Handa et al. [16 and 17] developed a Memectic algorithm within a geographic information system (GIS) that uses live weather reports to update the spreading routes. Actually, the system starts with previously designed routes, uses the weather forecast to update the road links that need to be serviced and then performs the Memetic algorithm. This algorithm, based on a genetic analogy, alters the previous routes (the parents) by exchanging selected route sections between the routes (crossovers) to create new ones (new generation). A local search is then performed by moving road links inside the routes or between routes. The new solution is then evaluated using a fitness evaluation which evaluates the cost of the routes and add penalty due to forbidden turns.

Omer [18] used a greedy randomized adaptive search procedure (GRASP) together with simulated annealing to design winter spreading routes. The first step, GRASP, is a constructive heuristic which build routes one by one, by randomly selecting road links from a candidate list. The road links in the list are selected in a way that the current route stays feasible and in order to satisfy a greedy evaluation function. The second step, the simulated annealing algorithm, is the improving phase. It takes the routes previously built and moves arcs between routes following given rules and applies repair operators to make the route feasible. At each step, a temperature evaluates the quality of the solution. The algorithm tends to decrease the temperature at each iteration to improve the quality of the solution. At some points, temperature increases are allowed in order to escape local minima which could prevent from getting a better solution. This algorithm performed well on theoretical benchmark tests, though no real life case study has been reported.

The dynamic aspect of real-life snow gritting routing was also addressed by Tagmouti et al. [19 and 20]. Their algorithm is based on a variable neighborhood descent which uses weather updates as input. The road segments are given with time windows during which they shall be serviced and the snow gritting routes are updated accordingly. This methodology was tested using theoretical instances as well. More recently, Salazar-Aguilar et al. [21] have studied the problem of snow plow synchronization. This case occurs when the road has many lanes that must be plowed simultaneously by several vehicles. To solve this problem, they designed an adaptive large neighborhood search heuristic. Realistic route sets were obtained on theoretical as well as a real-life instance.

3. METHODOLOGY

This section first presents the real-world case on which this study is based. This allows to overview the elements that have to be taken into account in the development of a route planning method. Afterwards, it proposes a mathematical model to address the case study constraints. The implementation phase is discussed in Section 4.

3.1. Case study

The case study is the municipality of Dolbeau-Mistassini, a city of 14 500 inhabitants located in the northern part of the province of Quebec. The road network of the municipality is 265 kilometres long, composed of residential streets, collectors, arterials, and rural roads. The map of the road network is presented in FIGURES 1 AND 2. TABLE 1 shows the characteristics of the road network. In average, the municipality receives annually 260 centimetres of snow during a period of about 6 months. The average temperature in January is -18°C.

The network shows an interesting road pattern mixing rural and urban characteristics. In the rural part, the street density is lower and most of the intersections are three-way junction. As opposed to the urban part where the street network is denser, there are more intersections and most of them are four-way junction.

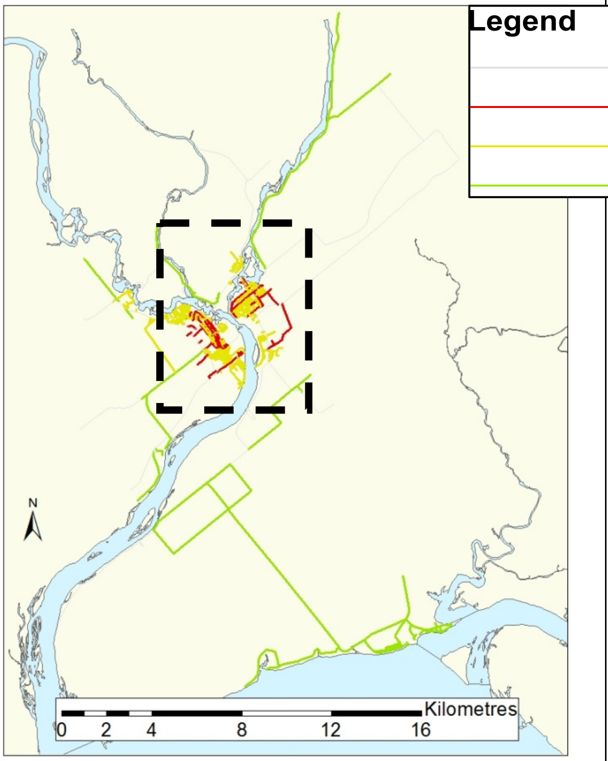

Figure 1 - Complete map of the case study area

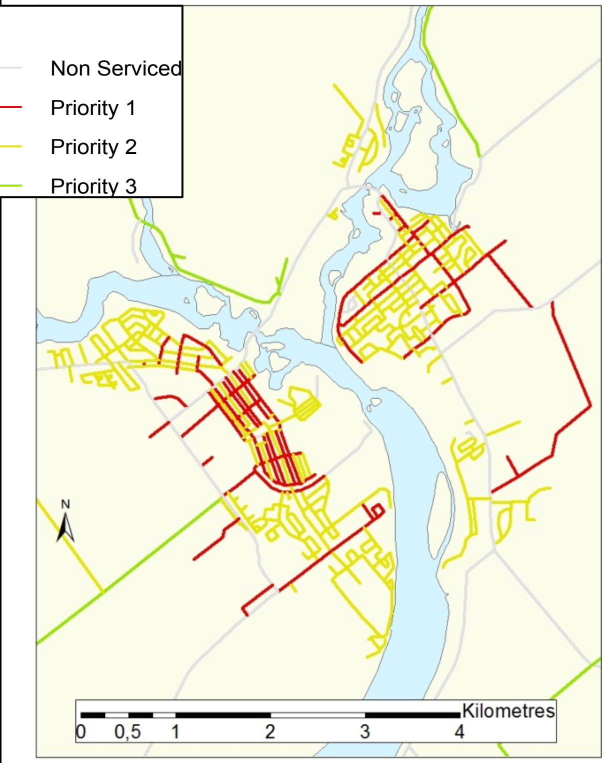

Figure 2 - Close-up of the urban part

Table 1 - Network characteristics of Dolbeau-Mistassini

Hierarchical priority	Roadway length (km)	Number of road links
1	27.5	198
2	66.9	483
3	84.4	105

The municipality is equipped with slow-speed front end loader for local streets, graders for more important urban streets and six wheeler trucks equipped for plowing and spreading. The speeds of the vehicle are given in TABLE 2 in function of the priority of the road segments. In this city, many constraints must be taken into account in the design of routes. In addition to those mentioned in the first section of this paper, the number of U-turns must be reduced, even avoided on major roads, to enhance the security of plowing operations. However U-turns are sometimes mandatory in the case of dead ends for example.

Table 2 - Speeds (km/h) of the vehicle according to the priority of the road segments.

	Veh 1	Veh 2	Veh 3	Veh 4	Veh 5	Veh 6	Veh 7	Veh 8
Priority 1	25	15	35	25	25	30	20	35
Priority 2	25	15	35	25	25	30	20	35
Priority 3	25	20	40	25	25	30	25	50
Not plowing	40	90	45	40	32	90	30	90

3.2. Mathematical formulation for the exact problem

The following mathematical formulation has been developed to model the problem and the constraints imposed by the case study.
The following variables are defined:

$$x_{ij}^{kvp} = \begin{cases} 1 & \text{if vehicle } v \text{ crosses the arc } i\text{, preceded by } j\text{, at position } k \text{ in priority class } p \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ij}^{kvp} = \begin{cases} 1 & \text{if vehicle } v \text{ services the arc } i\text{, preceded by } j\text{, at position } k \text{ in priority class } p \\ 0 & \text{otherwise} \end{cases}$$

t^{vp}	end time of priority p for vehicle$v$
$TMAX^P$	maximal end time for class priority$p$
T^{tot}	total of all end times of all vehicle routes

The following parameters are defined:

K	number of arcs per vehicle
V	number of vehicles
P	lowest priority (1 is the highest)
f_{ij}	penalty to turn from arc i to arc j.
a_{init}	an artificial arc that enters the depot (to permit a direct entrance in the network from the depot)
n^l	an artificial node that permit the exiting of the vehicle at any location to avoid penalties
M^{tot}	weighting factor for total time
M^P	weighting factor for the end of classes of priority

The following sets are defined:

A	set of arcs of the network (excluding artificial arcs)
A_{end}	set of arcs to reach $n^l$and to leave the network
A_{Aserv}^p	set of arcs to service, priority class p.
$j\|x_{ij}^{kvp}\exists$	j is the set of arcs where x_{ij}^{kvp}exists (ie there must be a link between i and j).

Objective:

$$1.Min\left(\begin{array}{c}\sum_{p=1}^{P}(TMAX^{P}M^{P})+T^{tot}M^{tot}\\ +\sum_{p=1}^{P+1}\sum_{v=1}^{V}\sum_{k=1}^{K}\sum_{i\in A}\sum_{j|x_{ij}^{kvp}\exists}f_{ij}\left(x_{ij}^{kvp}+y_{ij}^{kvp}\right)\end{array}\right)$$

Subject to:

2. $TMAX^{P}\geq t^{vp}\qquad p=\{1\ldots P,P+1\}$

3. $t^{vp}=\sum_{k=1}^{K}\sum_{i\in A}\sum_{j\in A}\left(x_{ij}^{kvp}tp_{i}^{v}+y_{ij}^{kvp}ts_{i}^{v}\right)\qquad p=\{1\ldots P,P+1\},v=\{1\ldots V\}$

4. $\sum_{p=1}^{P+1}\sum_{j\in A|x_{ij}^{kvp}\exists}\left(x_{ij}^{kvp}+y_{ij}^{kvp}\right)=\sum_{p=1}^{P+1}\sum_{l\in A|x_{li}^{k+1,vp}\exists}\left(x_{li}^{k+1,vp}+y_{li}^{k+1,vp}\right)$

$k=\{2\ldots K\},v=\{1\ldots V\},i\in A\cup A_{end}\cup a_{init}$

5. $\sum_{p=1}^{P+1}\sum_{j\in d}x_{ia_{init}}^{1vp}=1\qquad v=\{1\ldots V\}$

6. $\sum_{p=1}^{P+1}\sum_{i\in A}\sum_{j|x_{ij}^{1vp}\exists}x_{ij}^{1vp}=0\qquad v=\{1\ldots V\}$

7. $\sum_{k=1}^{K}\sum_{i\in A_{end}}\sum_{j\in A}x_{ij}^{kvP+1}=1\qquad v=\{1\ldots V\}$

8. $\sum_{j|x_{ij}^{kvp}\exists}\left(x_{ij}^{kvp}+y_{ij}^{kvp}\right)\leq\sum_{p*=p}^{P+1}\sum_{l|x_{li}^{kvp}\exists}\left(x_{li}^{k+1,vp*}+y_{li}^{k+1,vp*}\right)$

$k=\{2\ldots K\},p=\{1\ldots P,P+1\},i\in A\cup a_{init}\cup A_{end}$

9. $\sum_{k=1}^{K}\sum_{v=1}^{V}\sum_{p*=1}^{p}\sum_{j|y_{ij}^{kvp*}\exists}y_{ij}^{kvp*}=1\qquad i\in A_{ToBeServiced}^{p},p=\{1\ldots P\}$

10. $x_{ij}^{kvp}\in\{0,1\}\qquad i\in A\cup A_{end},j\in A\cup a_{init}\cup A_{end},k=\{1\ldots K\},v=\{1\ldots V\},$

$\mathrm{p}=\{1\ldots P,P+1\}$

11. $y_{ij}^{kvp}\in\{0,1\}$

$i\in A\cup A_{end},j\in A\cup a_{init}\cup A_{fin},k=\{1\ldots K\},v=\{1\ldots V\},p=\{1\ldots P,P+1\}$

12. $t^{vp}\geq 0\quad v=\{1\ldots V\},p=\{1\ldots P,P+1\}$

13. $TMAX^{p}\geq 0\qquad p=\{1\ldots P,P+1\}$

The objective function (equation 1) is composed of three parts:

1. $\sum_{p=1}^{P}(TMAX^{P}M^{P})$: aims to reduce the maximal route time of each class of priority p weighted by a factor M^{p}. In this case study, the following values were used: $M^{1}=1\,000$, $M^{2}=100$, $M^{3}=10$.
2. $T^{tot}M^{tot}$: aims to normalize the total of all routes time (T^{tot}) by a factor M^{tot}. In this case study, M^{tot}has been fixed to 5000 after several tries.
3. $\sum_{p=1}^{P+1}\sum_{v=1}^{V}\sum_{k=1}^{K}\sum_{i\in A}\sum_{j|x_{ij}^{kvp}\exists}f_{ij}\left(x_{ij}^{kvp}+y_{ij}^{kvp}\right)$: aims to penalize the turns with a factor f_{ij} related to the type of turn. FIGURE 3 shows that right turns are not penalized

because they do not leave a snow windrow on the street. If the vehicle goes straight ahead or turns left, it can cause obstruction on road lanes ($f_{ij} = 500$ and $f_{ij} = 5000$). For spreading operation, it can be preferable to favour straight lines instead of turns; penalty factor can then be adjusted. To select the turn penalty factor, the angle is calculated between two consecutive road links and is used as shown on FIGURE 3. The values of $\mathrm{M^p}$ and $\mathrm{f_{ij}}$ must be chosen according to the characteristics of the network.

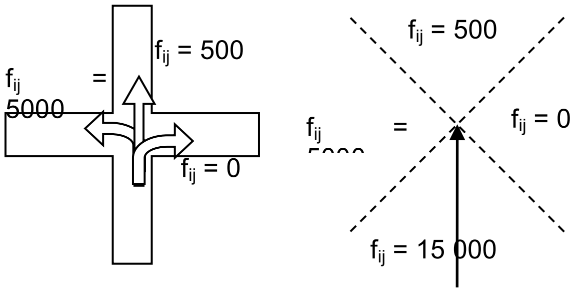

FIGURE 3 - Turning penalties

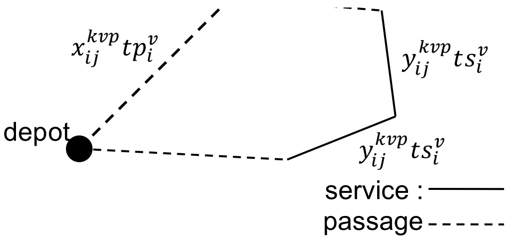

FIGURE 3 - Schematic representation of constraint (3).

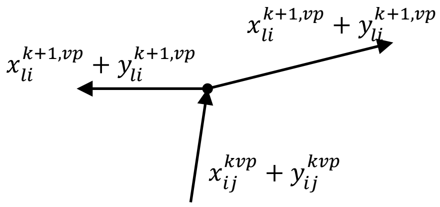

FIGURE 4 - Schematic representation of constraint (4).

The constraints defined in (2) find the ending time of each class priority p by selecting the highest t^{vp} among all vehicles. Constraints (3) sum all the service and crossing times to get the ending time for each priority and each vehicle (FIGURE 4). Constraints (4) ensure route continuity from k to $k+1$ for each possible class priority-vehicle (FIGURE 5). Constraints (5) and (6) ensure that all routes can only originate from the depot. To achieve this, all routes must have their $j = a_{init}$ at $k = 0$. a_{init} is an artificial arc that ends at the depot which precedes the first arc crossed or serviced for all routes. Constraints (7) allow vehicle v to leave the network at any k by taking an artificial link from the set A_{fin}. After this, the vehicle v arrives at the node n_l where no more turn, service or travelling penalty is incurred and it is impossible to go back into the network, therefore, the route is complete (FIGURE 6).

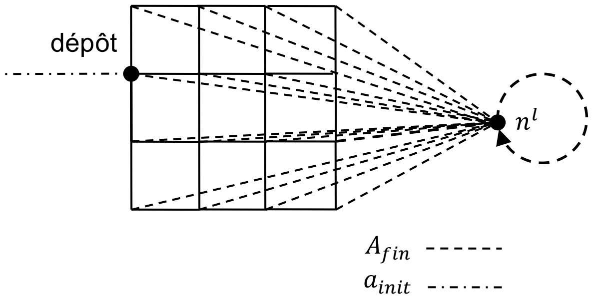

FIGURE 5 - Schematic representation of constraint (5) and (7).

Constraints (8) and (9) ensure continuity of the class priority p. All arcs that need to be serviced are categorized following their nominal priority in one of the sets $A^p_{ToBeServiced}$. On the other hand, a priority class p is ended when all arcs of $A^p_{ToBeServiced}$ are serviced. However, priority upgrade is possible when an arc of lower priority is serviced during a higher class priority at the cost of the higher priority. That means that all arcs of nominal priority *1*, must be serviced during class priority *1*. However, one arc of priority *2* (or lower) can be serviced during class priority *1*, though it will be penalized as an arc of priority *1* (FIGURES 5 and 6). Constraints (10), (11), (12) and (13) define the variables.

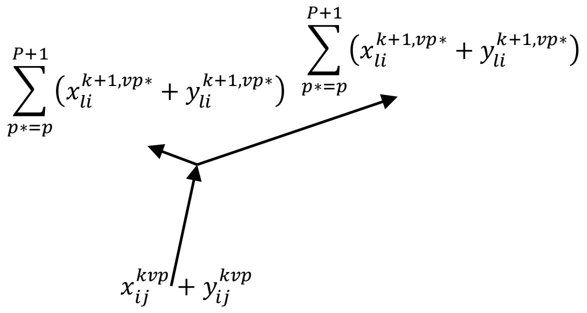

FIGURE 6 - Schematic representation of constraint (8).

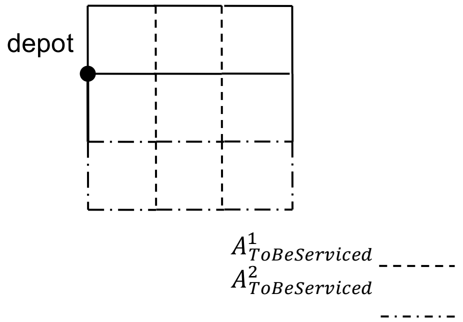

FIGURE 7 - Schematic representation of constraint (9).

4. IMPLEMENTATION

At this time, the implementation of the model on the real case road network cannot be done because of the millions of variables that would have to be put forward with the existing solver software.

A simple network has been designed to test the mathematical model (FIGURE 9). The parameters of the network are:

- road segment length or cost (t_i);
- road segment priority (p_i);
- position of the nodes to compute the turn penalties or the value of turn penalties directly from road segment j to i (f_{ij});
- position of vehicle depot, where all routes start;
- number of vehicles (V).

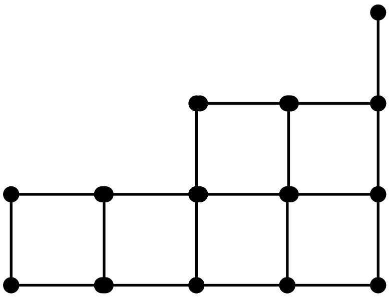

Figure 8 - Theoretical network used to test the linear formulation

The number of arcs in the networks has been kept low because of the high number of variables and constraints. It is mainly limited by K, the maximum number of road segments in each route. The resolution was limited to K=18 for two vehicles after what, the software could not ensure optimality. Indeed, even if the software does find an optimal solution, it does not insure the optimality of the solution since the number of arcs in each route is limited by the formulation.

This theoretical network is obviously not of the size of real case instances, which have typically more than 800 arcs, however it helped to ensure the applicability of the algorithm.

DISCUSSION AND CONCLUSION

This paper has presented an exact formulation for the real case study of the small town of Dolbeau-Mistassini, Quebec. The case study helped to identify operational constraints, namely turning penalties, vehicle capacity and speed, return to depot, hierarchy of the road network to be serviced, etc. The mathematical model can be solved, only however on a very small road network that does not correspond to a real case situation. This situation can be addressed in two ways: 1) use a powerful solver that could tackle the 800 arcs network; 2) develop a heuristics to create the snow plowing and deicing routes.

The development of a metaheuristics is to be privileged for further steps of this research project. In order to solve the case study, a near optimal approach is being developed. The algorithm is an Adaptive Large Neighborhood Search based on the one developed by Salazar-Aguilar et al. [21]. The advantage of using a metaheuristic is that it provides good routes when used on large instances, such as real-life cases. Furthermore, it can take into account several constraints required by the actual snow plowing operations while (hopefully) keeping the solution time relatively short. The methodology considers two phases: first, it creates a feasible solution (a set of feasible routes) using a simple construction heuristic. Then, the routes are improved by applying destruction/repair operators following predetermined rules. This application of operators demands a lot of experimentations. The objective function remains the same as the exact model; however the improvement phase of the metaheuristic can tolerate small deteriorations of the value of the objective, if it can help to reach a certain criterion of optimality. The resolution of the exact model, may it take days of computing time, could be helpful to compute the optimal value of the objective function. Hence, at the end, the results will still have to be validated by the snow plowing and deicing managers of the city. If they find some routes inapplicable, we could see why it is so and improve the heuristics to obtain better results.

Since our work is intended for practical use, it is worth saying that the results are available in a GIS format. That means that they can be easily converted to several other formats. The results can also be converted to a format supported by a GPS device used by truck drivers.

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